Lecture 16 Highlights
Phys 402

Spontaneous Emission and LASERs

A black body radiation spectrum has an energy density (energy per unit volume per unit frequency) given by Planck's formula:

$$\rho(\omega) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/k_B T} - 1}, \qquad (1)$$

where ω is the angular frequency of the radiation, k_B is Boltzmann's constant, and T is temperature of the blackbody radiator. Earlier we calculated the rate of absorption of this type of radiation by a two-level atom (states a and b) assuming random polarization of the light:

$$R_{a\to b} = \frac{\pi e^2 \rho(\omega_{ab})}{3\varepsilon_0 \hbar^2}\left(\left|x_{ab}\right|^2 + \left|y_{ab}\right|^2 + \left|z_{ab}\right|^2\right) \equiv \rho(\omega_{ab}) M_{ab},$$

where $E_b - E_a = \hbar\omega_{ab}$, and the term in parentheses contains the three Cartesian matrix elements between states a and b.

Upon examination of time dependent perturbation theory for the two-state atom, we have seen two processes leading to transitions in the atom. These processes are absorption (atom starts in the lower energy state, absorbs energy from the electromagnetic field, and makes a transition to the upper state), and stimulated emission (atom starts in the upper energy level, the electromagnetic field stimulates a transition to the lower energy state, and energy is added to the electromagnetic field). However if we apply these two processes to equilibrium conditions prevailing in the walls and electromagnetic fields of a "blackbody box," one finds that the rates of absorption and emission of the atoms cannot be balanced. Recognizing this problem, Einstein proposed an additional mechanism of emission, known as spontaneous emission. In this process an atom can relax from state 'b' to state 'a' by emitting a photon, but the process does not require a pre-existing photon to "tickle" the atom. Hence the rate at which this relaxation occurs is independent of the blackbody radiation energy density $\rho(\omega_{ab})$.

We can calculate the spontaneous emission rate A in a blackbody radiator in equilibrium. Let N_a be the number of atoms in the walls of the box in state a, and N_b is the corresponding number in state b. (We assume that the total number of atoms in the walls of the box $N = N_a + N_b$ is fixed.) The rate at which atoms join state b (the upper state) is given by a 3-term expression:

$$\frac{dN_b}{dt} = N_a M_{ab}\rho(\omega_{ab}) - N_b A - N_b M_{ba}\rho(\omega_{ab}),$$

where the matrix elements $M_{ba} = M_{ab}$ are symmetric. The terms on the right hand side represent absorption, spontaneous emission, and stimulated emission, respectively. In equilibrium we expect the number of atoms in state b to be unchanging, meaning $\frac{dN_b}{dt} = 0$. Solving for $\rho(\omega_{ab})$ yields:

$$\rho(\omega_{ab}) = \frac{A/M_{ab}}{e^{\hbar\omega_{ab}/k_B T} - 1}. \qquad (2)$$

To get this we assumed a Boltzmann distribution of occupation numbers of states b and a : $\frac{N_b}{N_a} = e^{-(E_b - E_a)/k_B T} = e^{-\hbar\omega_{ab}/k_B T}$. By comparing Eq. (2) to Eq. (1) for the Planck blackbody radiation formula, we can directly determine the spontaneous emission rate:

$$A = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{\pi e^2}{3\varepsilon_0 \hbar^2} \left(|x_{ab}|^2 + |y_{ab}|^2 + |z_{ab}|^2 \right).$$

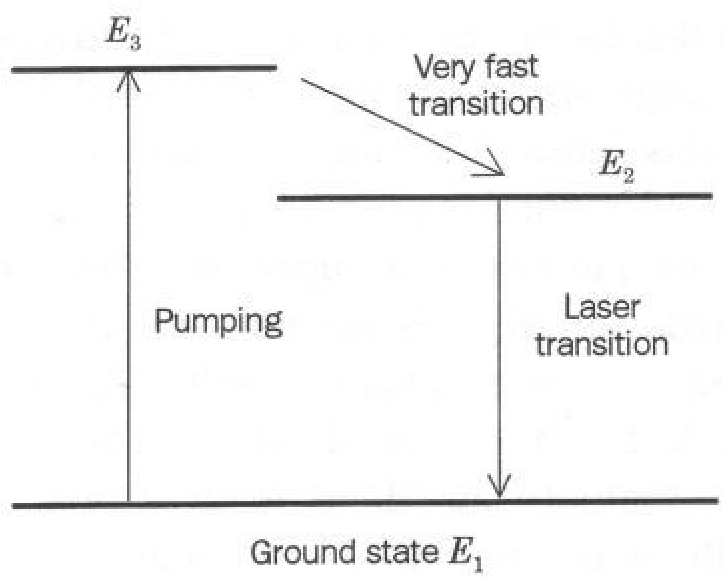
The spontaneous emission rate is independent of the energy density of radiation in the box, $\rho(\omega_{ab})$, and therefore the temperature of the walls of the box. The pre-factors are all quantum parameters. The matrix elements were inherited from the dipole approximation made in the original time-dependent perturbation calculation.

The spontaneous emission rate increases quickly with increasing energy difference between upper and lower states (ω^3). This limits the operation of lasers at high frequencies (beyond the visible range).

A LASER creates Light Amplification by Stimulated Emission of Radiation. It is basically a Xerox machine for photons. Three requirements must be met:

1) A pair of energy levels are needed for which a transition can take place.
2) A population inversion of the two states must be created. In other words a non-equilibrium situation in which there are more atoms in the upper state than the lower state must be created.
3) A resonance is required to keep the photons passing back and forth through the gain medium to allow the "Xeroxing" to take place by means of stimulated emission of radiation.

A typical laser uses 3 levels, as shown in the diagram:



Typically the pumping transition from E_1 to E_3 involves a $\Delta\ell = +1$ transition. The "very fast transition" shown in the diagram from E_3 to E_2 involves another $\Delta\ell = +1$ transition. The transition back to the ground state from E_2 to E_1 now involves a $\Delta\ell = -2$ transition which is dipole forbidden. Hence a population inversion will be created in energy level E_2. An atom in this state may undergo a spontaneous emission through a quadrupole process, for example (see HW 8 for those selection rules). This photon, if it stays inside the laser cavity, will then create other identical (in-phase) photons through stimulated emission. These photons will create other identical prodigy and there will be a chain reaction of identical photons built up inside the laser. This is the source of the intense coherent (phase coherent) light coming out of a laser. This process will continue until another spontaneously emitted photon is created and then takes over the lasing mode. This transition limits the "coherence length" of phase-coherent light that comes out of a laser. Coherence lengths of lasers can be in the kilometer range.

To summarize the steps in creating a laser:

1) The atoms are 'pumped' from E_1 to E_3 by means of electric dipole transitions with $\Delta\ell = +1$ (for example).
2) The atoms in energy level E_3 will either relax back to E_1 or go to E_2 with another $\Delta\ell = +1$ transition.
3) The atoms in E_2 can't get back to E_1 by an electric-dipole-allowed transition because $\Delta\ell = -2$. This is a forbidden transition, so the atoms are temporarily trapped there.
4) A single atom makes the transition from E_2 to E_1 by means of a rarer higher order process (e.g. electric quadrupole transition) thus creating a single 'mother photon' of energy E_2-E_1.
5) If the mother photon is emitted in the right direction it will bounce back and forth through the 'gain medium' (i.e. the region primed with lots of atoms in energy state E_2) and create many copies of itself through the stimulated emission process.
6) These "Xerox copied" photons are emitted in phase at the same frequency, building up a strong coherent light excitation in the optical resonator. Some of this energy is extracted through a partially reflecting mirror to create a coherent laser beam.
7) This is a highly simplified outline for how a laser works. Many people have found many creative ways to build lasers using gases, liquids, and solid state devices. The class web site has a brief summary of some of the complications associated with the He-Ne gas laser.

Historically the MASER (M is for microwave, $\hbar\omega \cong 10\ \mu eV$) was discovered first, in the 1950's. The MASER was thought of as an amplifier because it converted one photon into two identical photons moving in the same direction – quite a useful device. The idea of creating a LASER (L is for visible light, $\hbar\omega \cong 2\ eV$) was considered a major challenge because the spontaneous emission rate $A{\sim}\omega^3$ is about 18 orders of magnitude larger for visible light compared to microwaves. This makes it considerably harder to establish a population inversion for optical transitions. The tricks and quantum engineering outlined above helped to overcome this problem.

Fermi's Golden Rule

So far, our treatment of time-dependent perturbation theory has been applied to transitions from one discrete state to another discrete state. We then discussed the issue of time-dependent perturbation applied to a transition from a discrete state to a continuum of available states. For example, this applies to ionization of an atom, in which the electron is liberated from the atom and sent into a free-particle final state. Free particles have no quantization constraints, so there energies and momenta are continuously varying. One can introduce a Density of States (DOS) $g(E)$ which quantifies the number of available final states at energy E. The number of states between energy E and $E + dE$ is given by $g(E)\,dE$. For free particles, the density of states depends on the dimensionality of the space considered. The rate at which an atom will absorb energy from a harmonic electromagnetic perturbation and transition from discrete initial state "i" and go to a final state "f" at energy E_f in the continuum is given by $R_{i\to f} = \frac{2\pi}{\hbar} \left|\frac{\mathcal{H}_{if}'}{2}\right|^2 g(E_f)$. Here $\mathcal{H}_{if}'$ is the perturbing Hamiltonian matrix element. This is known as Fermi's Golden Rule.